Enlightenment of Euclid and Descartes on Geometry Teaching in Senior High School

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Abstract: This study, starting from the geometric perspective of mathematics in the senior high school, mainly elaborates on two critical contributors to geometry: Euclid and René Descartes, whose contributions and life stories provide frontline teachers with visual references for historical materials of mathematics.

Keywords: mathematics; geometry; Euclid; Descartes

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1. Introduction

Mathematics Curriculum Standards for Common Senior High School points out that "Mathematical culture is one of the most important content throughout the mathematics curriculum in the senior high school and is required to be incorporated into every section or topic [1]." The history of mathematics, an important constituent of mathematical culture, plays a key role in mathematics teaching in senior high school. As early as the nineteenth century, some mathematicians who were concerned with the history of mathematics had already realized the educational value of the history of mathematics. For example, the pioneer of History & Pedagogy of Mathematics (HPM) and an American historian of mathematics Florian Cajori compiled many works, such as the Teaching and History of Mathematics in the United States and the History of Mathematics, and elaborated the educational value of the history of mathematics from two aspects of "it makes the discipline more attractive" and "the history of one discipline is the teaching guide for the discipline" [2]. With the extensive and in-depth exploration of the educational value of the history of mathematics, the development of the relationship between HPM as a field of academic research was marked in 1972 established by the International Research Team at the Second International Congress on Mathematical Education. Since the HPM took shape, a historian of mathematics Leonard Richard Fowell summarized fifteen reasons for applying the history of mathematics to mathematics teaching; C. Tzanakis and A. Arcavi provided a clearer classification of the educational value of the history of mathematics in terms of mathematics learning, nature of mathematics and mathematics activities, background knowledge of the teacher, mathematics emotion, and mathematics as a cultural activity. Subsequently, more and more educationalists have devoted themselves to the theoretical study of HPM. It has been well documented that the educational value of the history of mathematics is now recognized not only by many educationalists but also by frontline teachers. From the conceptual perspective, the knowledge about the history of mathematics acquired by the teachers not only enriches their knowledge reserve and teaching resources but also helps them to better understand the nature of mathematics discipline; The knowledge about the history of mathematics can help students to understand mathematics disciplines, broaden their mind, and cultivate the "non－logical" thinking. However, many problems have been found in the geometry teaching practice of senior high school, such as students' lack of intense interest in learning, single teaching approaches, and lack of discussion and communication in the classroom [3]. Therefore, it seems urgent and necessary to efficiently implement the geometry curriculum and develop students' abilities of mathematical abstraction and intuitive imagination under the core literacy. This study aims to present the key knowledge about the history of mathematics in geometry to teachers and students, to enrich the historical materials, and to promote their understanding of geometry.

2. Geometry Development

Geometry, as one of the ancient branches of mathematics discipline [4], originates from four ancient civilizations [5]. Early geometry arose for the needs of production practice and was a collection of empirical laws concerning length, angle, area, and volume. But the geometric knowledge was used only as a practical tool rather than an independent discipline. The earliest known geometry texts were found from the Egyptian Rhind Papyrus (2000－1800 B. C.) and the Moscow Mathematical Papyrus (c. 1890 B. C.), and the ancient Babylonian Plimpton 322 (1900 B. C.). Around 400 B. C., ancient China also accumulated a lot of geometric knowledge, notably in the old mathematics books － Chou Pei Suan Ching, and the Mohist classic － Mohist Canon, etc. Building on the basic knowledge about experimental geometry in ancient Egypt and ancient Babylon, ancient Greek mathematician Euclid (c. 330－275 B. C.) used logical reasoning to elevate the study of geometry to a high degree of systematization and rigorousness. At that point, his works Elements of Geometry marked the birth of the axiomatization of a mathematical theory, and geometry became an independent discipline. As Albert Einstein said, "Here for the first time the world witnessed the miracle of a logical system which proceeded from step to step with such precision that every single one of its propositions was absolutely indubitable [5]." For a long time afterward, the strict deductive system became a shining example of geometry. In the 16th century, astronomy, navigation and other aspects popped up new needs for geometry with the rapid development of production and science, and technology, and Algebra was in the flourishing stage. At this time, Descartes realized that algebra could serve as the foundation for a discipline. He emphasized the universality of algebra and absorbed all essence in algebra and geometry [6]. In the 17th century, a branch of geometry that uses the algebraic approach to study the relationships among geometric objects and their properties was developed － analytic geometry. Generally speaking, classical geometry has undergone two significant breakthroughs in its development history: the establishment of the axiomatization system in Euclid's Elements of Geometry and the creation of analytic geometry by Descartes. Nowadays, geometry has several sub－disciplines, including plane geometry, solid geometry, non－euclidean geometry, and analytic geometry. This study selects solid geometry and analytic geometry from geometry in senior high school and fully explores the scientific and cultural ideas therein.

3. Solid Geometry and Analytic Geometry

3.1 Solid geometry in the textbook

Solid geometry is the study of the properties of spatial figures by logical reasoning from the basic theorems and concepts and based on different variations of various geometric figures [7]. It aims to mainly solve the problems such as the plane and perpendicularity, angles, and distances between spatial statistics. The content of solid geometry in PEP mathematics textbooks is primarily distributed in Chapter I Space Geometry and Chapter II Position Relations among Points, Lines, and Planes of Compulsory 2 and Chapter III Spatial Vectors and Solid Geometry of Elective 2－1. Chapter I Space Geometry of Compulsory 2 begins with three sections on the structure, three views and visualizations, and surface area and volume of space geometry. The textbook consciously presents the knowledge about the history of mathematics and briefly introduces the origin and development of descriptive geometry, the life story of the father of descriptive geometry, Gaspard Monge, and Zu Geng principle; Chapter II Position Relations among Points, Lines, and Planes begins with three sections on the relations among points, lines, and planes in space, the determination theorems and properties of parallel lines and planes, and the determination and properties of perpendicular lines and planes. In this chapter, the textbook introduces Euclid's Elements of Geometry and the axiomatization method on one page. Chapter III Spatial Vectors and Solid Geometry of Elective 2－1 begins with two sections on spatial vectors and their operations and vector methods in the space geometry; such sections do not cover the history of mathematics. In general, the section on solid geometry in the textbook covers three historical records, all of which are presented as reading material and are mainly used in an additive manner. Teachers can appropriately apply these historical records and flexibly use a variety of integration methods to give full play to the educational value of the history of mathematics in the classroom. The following sections expand and enrich the history of mathematics related to solid geometry from the historical background and axiomatization idea of Euclid.

Fig. 1

3.1.1 Historical background of Euclid. Beginning in the 6th century A. D., Greece ushered in the European culture trend in which mathematics was highly active. Many brilliant mathematicians appeared at that time, such as Thales, Pythagoras, and Plato. After 300 years of prosperity, the riots in the internal city－states of Greece led to a war across Athens and Sparta, which lasted for 30 years, and the northern Macedonians took advantage of the situation and conquered Athens with internal and external problems. Athens' economy and culture suffered a lot, and its prosperous times were in decline. The previous vibrant cultural center began to make its way across the Mediterranean to the newly emerging port of Alexandria, which also yearned for Greek civilization. Then, King Ptolemy established the Alexandria Museum and Library to invite scholars worldwide to come here for academic exchange and discussion. Its members got the subsistence and food expenses and were exempt from income tax, and these benefits attracted many scholars from Greece, Euclid among them. He sailed across the ocean, taking a ship from Athens to Alexandria. More and more scholars gathered, and slowly the museum and library became the academic research center for the

Greek humanities and natural sciences. These members also transferred from learning to teaching. It is said that Euclid was well respected by his students while teaching at Alexandria University and that he was always kind to those who loved mathematics and worked hard, and strict in his advice and criticism of those who were impatient for quick results. Once, Euclid was giving a lecture to his students. A newly enrolled student asked Euclid how geometry could be beneficial in life. Euclid looked calm and ordered the servant beside him, saying, "Give him three gold coins, since he wants to obtain profits from the study of geometry." There is also a story recorded in Proclus′s Summary of Geometry Development. King Ptolemy asked Euclid if there's any shortcut to the study of geometry other than the Elements of Geometry. Euclid replied forcefully, "There was no royal road to geometry." Euclid devoted his entire life to the study of geometry, and his pure love for mathematics and persistence in and pursuit of reason are worthy of being studied by everyone.

3.1.2 Elements of Geometry — Euclid. Euclid′s outstanding contribution was to combine these fragmentary, ancient mathematical ideas with his personal wisdom to create the axiomatization system of Euclidean geometry. Euclid realized that numerous theorems and conclusions were based on some general findings and that these known conclusions had their more basic theories. In other words, every theorem and conclusion is supported only by one or a few of the most basic ideas in the process of layer-by-layer reasoning, and these basic conclusions are apparent truths that do not require strict mathematical justification. Euclid named these most basic conclusions applicable to all mathematics sub-disciplines as axioms, those applicable to geometry as postulates [8]. The Elements of Geometry contains 13 volumes, covering five axioms, five postulates, 119 definitions, and 465 propositions, as shown in Table 1.

Table 1 Main Content in Each Volume of Elements of Geometry

Volume	Number of topics	Main content
I	48	Properties of point, line, plane, and other figures
II	14	Area conversion
III	37	Study on circle
IV	16	Inscribed and circumscribed polygon
V	25	Theory of proportions
VI	33	The theory of proportions is applicable to similar figures
VII	39	Proportions of sub-multiple, multiple, and integer
VIII	27	Geometric progression, continued proportion, square number, and cubic number
IX	36	Plane number, stereo number, prime number, an odd number, even number, and perfect number
X	115	Immeasurability
XI	39	Properties of solid figures
XII	18	Theory of area and volume
XIII	18	Regular polygon and regular polyhedron

Five axioms: ① Things which are equal to the same thing are also equal to one another. ② If equals be added to equals, the wholes are equal. ③ If equals be subtracted from equals, the remainders are equal. ④ Things which coincide with one another are equal to one another. ⑤ The whole is greater than the part. Five postulates: ① To draw a straight line from any point to any point. ② To produce a finite straight line continuously in a straight line. ③ To describe a circle with any center and distance. ④ That all right angles are equal to one another. ⑤ That if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the straight lines, if produced indefinitely, will meet on that side on which the angles are less that two right angles. If we compare the basic content of the Elements of Geometry with our current high school mathematics textbooks, we can find that the mathematics knowledge in the junior high school is mainly based on the content of the first six volumes, and the mathematics knowledge in the senior high school is mainly based on the content of the last seven volumes[9]. The Elements of Geometry was an outstanding achievement of ancient Greek mathematics and a milestone advancement in the history of mathematics and had a profound impact on the preparation of mathematics textbooks in China. It can be carefully found that some statements are incorrect, such as Postulate 2, the statement about "A finite straight line" is not precise, as a straight line can be extended to both ends infinitely. But this does not affect the historical status of the Elements of Geometry, which is considered to be the earliest example of a deductive system established based on the axioms due to its rich content and strict demonstration.

3.2 Analytic geometry

3.2.1 Descartes' ideological implication of analytic geometry. Analytic geometry is a discipline that uses the algebraic approach to study plane geometry problems. Analytic geometry of senior high school mathematics mainly includes the space rectangular coordinate system (Compulsory 2), the equations of lines and circles (Compulsory 2), the conic curves (Elective 2—1), the parametric equations and polar coordinates (Elective 4—4), etc. These parts are the expansion and enhancement of plane geometry in junior high school and the foundation of space analytic geometry and calculus in college mathematics. The creation of analytic geometry is one of the important achievements of mathematics in the 17th century, which is of epoch—making significance in the development history of mathematics. It contains the unique mathematical thought, spirit, and methods of Descartes. From the perspective of mathematical culture, his ideological implication is mainly reflected in the following six aspects: ① Historical origin: cultural renaissance and rapid development of productive forces, and higher requirements for mathematics and technology; ② Mathematical structure: the mathematical structure of Descartes' analytic geometry consists of the core concept, basic method, and mathematical principles. The core concepts are curves and equations, the basic method is to algebraize geometric problems and geometrize the algebraic problems, and the mathematical principle is the mapping principle; ③ Scientific value: The concepts of variables and coordinates were introduced into mathematics, creating a precedent for modern mathematics; ④ Philosophical performance: Generalized scientific method and universal philosophical method; ⑤Cognitive model: The thinking clues for problem solution follow the order of development, i.e., intuitive thinking → abstract thinking → deductive thinking; ⑥ Personality traits: The high—quality spirit of questioning, criticizing, and constant thinking[10].

3.2.2 "Cardioid" Descartes. Descartes studied many beautiful curves throughout his life, such as spiral curves, butterfly curves, and cardioid. The section on calculating the area of the curved figure using definite integrals can be explained by introducing Descartes' romantic love story to add a vibrant color. Descartes was born in Indre, France, in 1596, and once begged for food in the streets of the Swedish capital under the strain of poverty. Because of his accent, Descartes attracted the attention of the Swedish princess Christine who was keen on mathematics. She became very interested in Descartes and invited him to serve as her mathematics teacher. Under the careful guidance of Descartes, Christine got better and better at mathematics. As time passed, the sparks of love flew between them. But the love between teacher and student was vigorously opposed by King. He ordered to execute Descartes, but Christine threatened King with her life. King finally compromised for his love to his daughter, expelling Descartes back to France and placing Christine under house arrest. Since then, Descartes had to write to Christine about his feeling of missing. But his letters were repeatedly intercepted by King and never received by Christine, and he never received a reply. Shortly afterward, Descartes went down with the Black Death and sent his thirteenth letter before dying. There is only one short mathematical formula ""in this letter. Christine's father wanted to know the meaning of this formula, so he summoned the city's mathematicians to the palace to solve the mystery, but no one could solve the formula. So, King thought it was just a simple letter of academic exchange and gave this letter to Christine. Christine immediately understood Descartes' intention after reading it because only she and Descartes knew the polar coordinate system. She used the coordinate system taught by Descartes to draw the image corresponding to the formula (as shown in Figure 2). This is the famous "cardioid," and she was thrilled to realize that Descartes had never stopped loving her. The letter is still preserved in the European Descartes Memorial. Their story shows us the mathematician's romantic and poignant love story and an unusual and unique love letter of confession. The simple mathematical formula contains Descartes' immense wisdom and rich emotions, making the world shock.

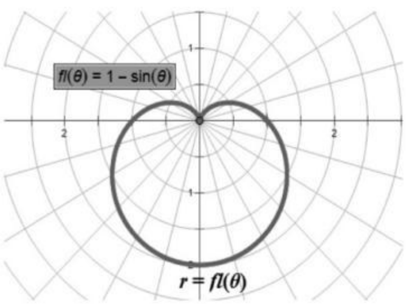

Fig. 2

4. Enlightenment

Understanding the history of geometry can broaden teachers' horizons and add new materials to their teaching. The following aims to elaborate the enlightenment of this study from two aspects especially.

4.1 Recognize the educational value of classical geometry

The development of classical geometry has undergone the process of experimental geometry → reasoning → algebraic calculation. Its embodied axiomatization system and the idea of symbolic — graphic combination have become one of the thinking methods for modern mathematics. Its thinking methods not only were penetrated into various branches of mathematics but also became one of the basic research methods for natural and social sciences and had the general methodological guidance value for the research of other sciences. Teachers should carefully read the Elements of Geometry to grasp its embodied thinking methods, subliminally cultivate the spirit of the rationality of students, and implement the cultivation of core mathematical literacy; According to the principle of historical similarity, the difficulties that mathematicians encountered in mathematical activities are likely to be the same difficulties that students will face; Understanding the development history of classical geometry will help teachers foresee the mistakes that their students may make, grasp the learning situation, and also allow the teachers have a stronger empathy for their students; The stage theory of human cultural history states that the sequence of children's psychological development replicates that of human civilization history[11]. Therefore, teachers should try to keep the sequence of students' psychological development stages and geometry development consistent to facilitate their understanding when teaching geometry.

4. 2 Emphasize the exploration and penetration of historical materials of mathematics

A few historical materials of mathematics are presented in the geometry section of the PEP senior high school mathematics textbook. All of them are in the form of reading material, which makes it difficult to stimulate the students' genuine interest in the discipline. When teaching solid geometry and analytic geometry, teachers can select the materials of this study to enrich the historical materials. For example, Euclid's historical background can be introduced in the teaching process of solid geometry to show his pure love and relentless pursuit of mathematics; The presentation of propositions referred to in the Elements of Geometry and feeling of axiomatization system can also let students see that these famous mathematicians are imprecise in the representation of certain concepts, which will enhance their confidence in learning mathematics and the courage to overcome difficulties. The six significant ideological implications can be incorporated into the teaching of analytic geometry to allow students to experience the mathematical ideas in analytic geometry, i. e. , the algebraic methods are used to study the geometric problems. Students can realize the mathematical problems with the changing and developing perspectives in the learning process. The romantic and poignant love story of Descartes and Princess Christine can make students appreciate the humanistic value and emotional care behind mathematics. In conclusion, the value of the history and education of mathematics stands up to the scrutiny and practice and is urgent to be practiced by every educator.

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